# AGNesF: Adaptive Gaussian Nested Filter for Parameter Estimation and State Tracking in Dynamical Systems

## Background: nested methodology

**Goal**: Computation of the joint posterior pdf

$$p(\boldsymbol{\theta}, \boldsymbol{x}_t | \boldsymbol{y}_{1:t}) = \underbrace{p(\boldsymbol{x}_t | \boldsymbol{\theta}, \boldsymbol{y}_{1:t})}_{\text{bottom layer}} \times \underbrace{p(\boldsymbol{\theta} | \boldsymbol{y}_{1:t})}_{\text{top layer}}$$

- $\boldsymbol{\theta}$ are static parameters
- $\boldsymbol{x}_t$ is a dynamic state variable

**Structure**:

Top layer: $\boldsymbol{\theta}$ estimation.

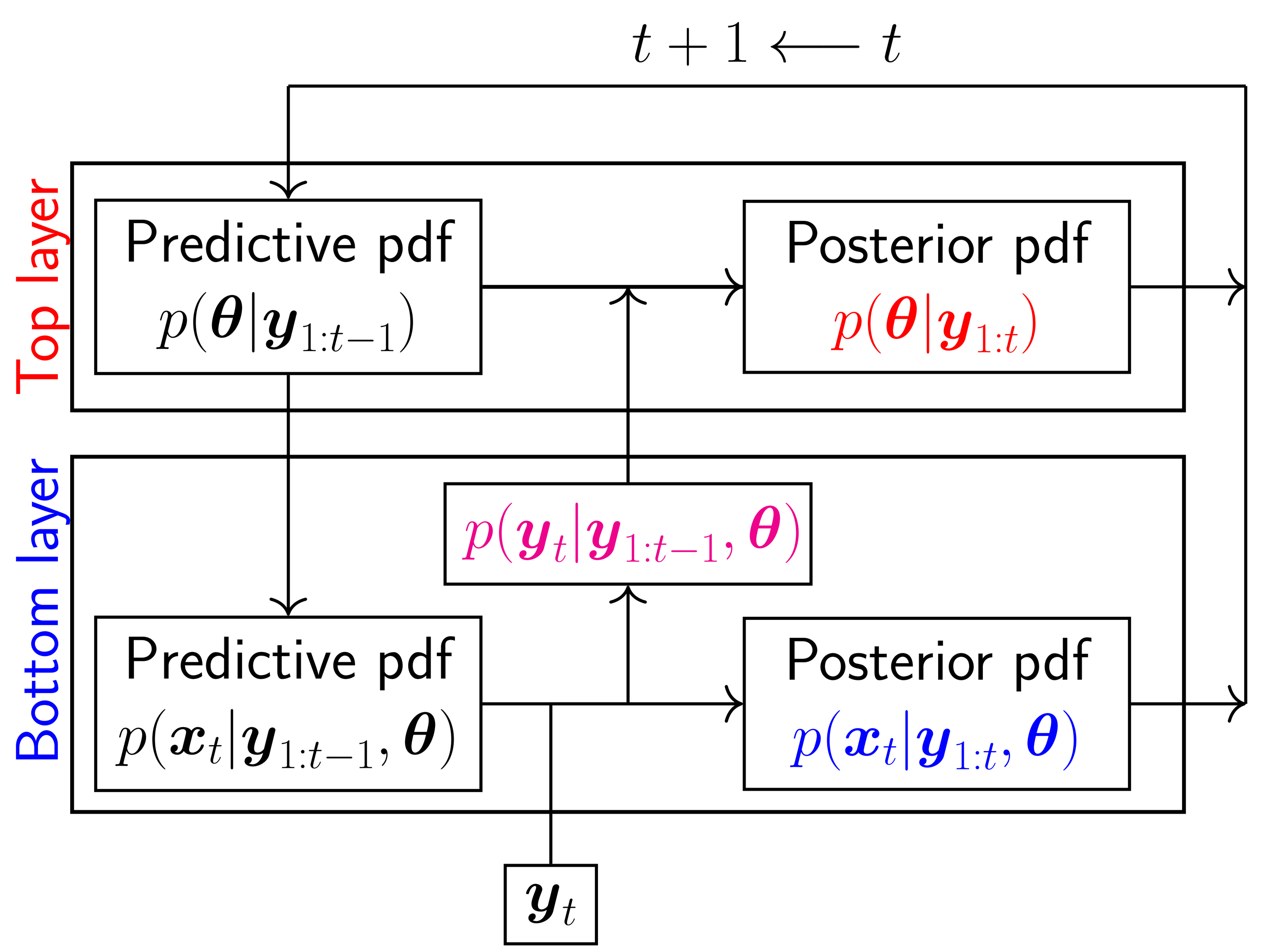
- Use of sampling filtering techniques, e.g., SMC, SQMC, UKF, QKF.
- $p(\boldsymbol{\theta}|\boldsymbol{y}_{1:t})$ is represented with $\{\boldsymbol{\theta}_t^i, w_t^i\}_{i=1}^{N_\theta}$.

Bottom layer: $\boldsymbol{x}$ tracking.

- Use of any filtering technique, e.g., SMC, EKF, UKF.
- Implementation of $N_\theta$ filters (one for each $\boldsymbol{\theta}_t^i$).

Key point: likelihood computation.

- $p(\boldsymbol{y}_t|\boldsymbol{y}_{t-1}, \boldsymbol{\theta})$ is computed in the bottom layer.




## Objective and approach

Objective:

- **Reduce computational complexity** of nested Gaussian filters without compromising performance.

Approach:

- **Reduce the number of points**, $N_\theta$, when parameters are close to convergence.
- Decision based on an **adaptive rule**.

## The statistic $\rho_t$

The **sample quality** is assessed with:

$$\rho_t = \frac{1}{\sum_{n=1}^{N_{\theta,t}} (\bar{s}_t^n)^2} \qquad \text{with} \qquad \bar{s}_t^n = \frac{p(\boldsymbol{y}_t|\boldsymbol{y}_{1:t-1}, \boldsymbol{\theta}_t^n)}{\sum_{n=1}^{N_{\theta,t}} p(\boldsymbol{y}_t|\boldsymbol{y}_{1:t-1}, \boldsymbol{\theta}_t^n)}.$$

- It takes its **minimum value** in $\rho_t = 1$, when only one $p(\boldsymbol{y}_t|\boldsymbol{y}_{1:t-1}, \boldsymbol{\theta}_t^n)$, is different from zero.
- It takes its **maximum value** in $\rho_t = N_{\theta,t}$, when for all the evaluations $p(\boldsymbol{y}_t|\boldsymbol{y}_{1:t-1}, \boldsymbol{\theta}_t^n)$ are equal.

$\longrightarrow$ Similar to ESS, but with different interpretation.

## Adaptive rule

**Fixed $N_\theta$.** Using a QKF in the top layer, $N_\theta$ is computed as

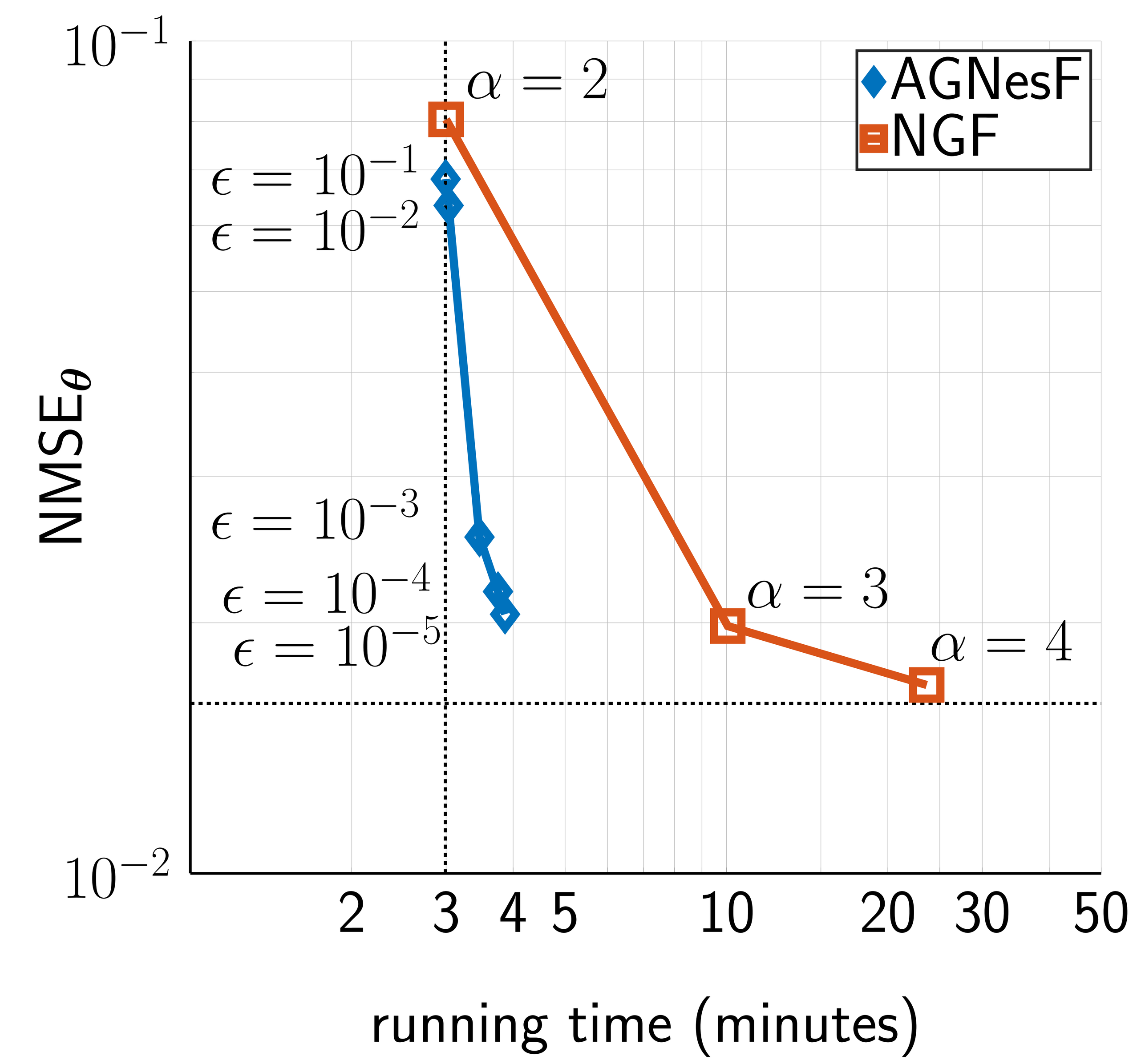
$$N_\theta = \alpha^{d_\theta}.$$

**Adaptive $N_{\theta,t}$.** We change $\alpha$ over time:

- If $\frac{\rho_t}{N_{\theta,t}} < 1 - \epsilon$,
$$N_{\theta,t+1} = \alpha_{t+1}^{d_\theta} \quad \text{with} \quad \alpha_{t+1} = \max(\alpha_t - 1, \alpha_{\text{min}}).$$
- Otherwise, $N_{\theta,t+1} = N_{\theta,t}$.

## Numerical Experiments

- Synthetic data of **Lorenz 63 model**.
- Estimation of $\boldsymbol{x}_t$ and $\boldsymbol{\theta} = [S, R, B]^\top$.
- Comparison of:
  - AGNesF with $\alpha_0 = 4$ and $\alpha_{\text{min}} = 2$.
  - Nested Gaussian filter (NGF) with fixed $\alpha$ and $N_\theta$.




## More details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